

## Supplementary materials for “Finite-sample adjustments in variance estimators for clustered competing risks regression” by Chen and Li

**WEB TABLE 1** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.5$ . Number of cluster  $n \in \{8, 10, 20, 30\}$  and cluster size  $\bar{m} \in \{10, 20, 50\}$ .

Parameter	$\bar{m}$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
				ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	8	10	0.536	0.349	-34.9	0.628	17.2	0.420	-21.6	0.412	-23.1	0.416	-22.4
		20	0.524	0.335	-36.1	0.507	-3.2	0.397	-24.2	0.395	-24.6	0.395	-24.6
		50	0.501	0.318	-36.5	0.479	-4.4	0.377	-24.8	0.375	-25.1	0.372	-25.7
	10	10	0.418	0.308	-26.3	0.428	2.4	0.355	-15.1	0.353	-15.6	0.352	-15.8
		20	0.409	0.297	-27.4	0.446	9.0	0.342	-16.4	0.339	-17.1	0.337	-17.6
		50	0.415	0.296	-28.7	0.405	-2.4	0.340	-18.1	0.340	-18.1	0.334	-19.5
	20	10	0.261	0.222	-14.9	0.258	-1.1	0.239	-8.4	0.238	-8.8	0.237	-9.2
		20	0.245	0.211	-13.9	0.245	0.0	0.227	-7.3	0.226	-7.8	0.224	-8.6
		50	0.239	0.206	-13.8	0.239	0.0	0.221	-7.5	0.221	-7.5	0.218	-8.8
	30	10	0.193	0.177	-8.3	0.196	1.6	0.186	-3.6	0.186	-3.6	0.184	-4.7
		20	0.199	0.172	-13.6	0.190	-4.5	0.180	-9.5	0.180	-9.5	0.178	-10.6
		50	0.190	0.168	-11.6	0.185	-2.6	0.176	-7.4	0.176	-7.4	0.174	-8.4
$\beta_{12}$	8	10	0.189	0.154	-18.5	0.198	4.8	0.172	-9.0	0.171	-9.5	0.223	18.0
		20	0.145	0.117	-19.3	0.148	2.1	0.131	-9.7	0.130	-10.3	0.183	26.2
		50	0.110	0.092	-16.4	0.116	5.5	0.102	-7.3	0.102	-7.3	0.158	43.6
	10	10	0.171	0.139	-18.7	0.169	-1.2	0.152	-11.1	0.151	-11.7	0.189	10.5
		20	0.125	0.109	-12.8	0.130	4.0	0.118	-5.6	0.117	-6.4	0.156	24.8
		50	0.097	0.085	-12.4	0.100	3.1	0.092	-5.2	0.092	-5.2	0.135	39.2
	20	10	0.111	0.103	-7.2	0.113	1.8	0.108	-2.7	0.107	-3.6	0.120	8.1
		20	0.086	0.080	-7.0	0.087	1.2	0.083	-3.5	0.083	-3.5	0.097	12.8
		50	0.066	0.063	-4.5	0.068	3.0	0.065	-1.5	0.065	-1.5	0.081	22.7
	30	10	0.085	0.085	0.0	0.090	5.9	0.088	3.5	0.087	2.4	0.094	10.6
		20	0.068	0.066	-2.9	0.069	1.5	0.067	-1.5	0.067	-1.5	0.075	10.3
		50	0.052	0.051	-1.9	0.053	1.9	0.052	0.0	0.052	0.0	0.061	17.3

**WEB TABLE 2** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under an unbalanced design with  $CV = 0.5$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.9$ . Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} \in \{10, 20, 50\}$ .

$\beta$	$\bar{m}$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
				ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	8	10	0.321	0.208	-35.2	0.355	10.6	0.252	-21.5	0.246	-23.4	0.265	-17.4
		20	0.284	0.177	-37.7	0.279	-1.8	0.213	-25.0	0.211	-25.7	0.219	-22.9
		50	0.250	0.157	-37.2	0.284	13.6	0.193	-22.8	0.189	-24.4	0.189	-24.4
	10	10	0.267	0.187	-30.0	0.271	1.5	0.218	-18.4	0.217	-18.7	0.225	-15.7
		20	0.236	0.166	-29.7	0.235	-0.4	0.193	-18.2	0.192	-18.6	0.194	-17.8
		50	0.214	0.144	-32.7	0.210	-1.9	0.169	-21.0	0.168	-21.5	0.166	-22.4
	20	10	0.173	0.139	-19.7	0.167	-3.5	0.151	-12.7	0.151	-12.7	0.150	-13.3
		20	0.146	0.119	-18.5	0.142	-2.7	0.129	-11.6	0.129	-11.6	0.128	-12.3
		50	0.137	0.109	-20.4	0.131	-4.4	0.118	-13.9	0.119	-13.1	0.116	-15.3
	30	10	0.128	0.114	-10.9	0.130	1.6	0.121	-5.5	0.121	-5.5	0.120	-6.3
		20	0.122	0.102	-16.4	0.115	-5.7	0.108	-11.5	0.108	-11.5	0.107	-12.3
		50	0.104	0.092	-11.5	0.104	0.0	0.098	-5.8	0.098	-5.8	0.096	-7.7
$\beta_{12}$	8	10	0.186	0.157	-15.6	0.204	9.7	0.176	-5.4	0.175	-5.9	0.206	10.8
		20	0.130	0.107	-17.7	0.134	3.1	0.119	-8.5	0.118	-9.2	0.145	11.5
		50	0.090	0.072	-20.0	0.092	2.2	0.080	-11.1	0.080	-11.1	0.104	15.6
	10	10	0.169	0.139	-17.8	0.169	0.0	0.152	-10.1	0.151	-10.7	0.172	1.8
		20	0.125	0.101	-19.2	0.122	-2.4	0.110	-12.0	0.110	-12.0	0.128	2.4
		50	0.081	0.068	-16.0	0.081	0.0	0.074	-8.6	0.073	-9.9	0.091	12.3
	20	10	0.113	0.102	-9.7	0.113	0.0	0.107	-5.3	0.106	-6.2	0.113	0.0
		20	0.083	0.075	-9.6	0.083	0.0	0.079	-4.8	0.078	-6.0	0.085	2.4
		50	0.059	0.051	-13.6	0.056	-5.1	0.053	-10.2	0.053	-10.2	0.060	1.7
	30	10	0.091	0.085	-6.6	0.092	1.1	0.088	-3.3	0.088	-3.3	0.091	0.0
		20	0.064	0.062	-3.1	0.066	3.1	0.064	0.0	0.064	0.0	0.067	4.7
		50	0.047	0.044	-6.4	0.047	0.0	0.045	-4.3	0.045	-4.3	0.049	4.3

**WEB TABLE 3** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under an unbalanced design with  $CV = 0.5$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.5$ . Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} \in \{10, 20, 50\}$ .

$\beta$	$\bar{m}$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
				ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	8	10	0.538	0.354	-34.2	0.632	17.5	0.433	-19.5	0.424	-21.2	0.423	-21.4
		20	0.591	0.345	-41.6	0.948	60.4	0.432	-26.9	0.415	-29.8	0.407	-31.1
		50	0.507	0.333	-34.3	0.832	64.1	0.419	-17.4	0.398	-21.5	0.390	-23.1
	10	10	0.460	0.320	-30.4	0.467	1.5	0.375	-18.5	0.373	-18.9	0.367	-20.2
		20	0.450	0.312	-30.7	0.467	3.8	0.367	-18.4	0.363	-19.3	0.354	-21.3
		50	0.429	0.297	-30.8	0.443	3.3	0.353	-17.7	0.345	-19.6	0.335	-21.9
	20	10	0.282	0.229	-18.8	0.273	-3.2	0.249	-11.7	0.248	-12.1	0.244	-13.5
		20	0.256	0.222	-13.3	0.265	3.5	0.242	-5.5	0.242	-5.5	0.236	-7.8
		50	0.255	0.216	-15.3	0.260	2.0	0.236	-7.5	0.236	-7.5	0.229	-10.2
	30	10	0.220	0.189	-14.1	0.215	-2.3	0.201	-8.6	0.201	-8.6	0.197	-10.5
		20	0.212	0.185	-12.7	0.211	-0.5	0.197	-7.1	0.197	-7.1	0.193	-9.0
		50	0.209	0.183	-12.4	0.208	-0.5	0.195	-6.7	0.195	-6.7	0.190	-9.1
$\beta_{12}$	8	10	0.204	0.159	-22.1	0.215	5.4	0.181	-11.3	0.179	-12.3	0.228	11.8
		20	0.155	0.121	-21.9	0.175	12.9	0.137	-11.6	0.136	-12.3	0.187	20.6
		50	0.117	0.095	-18.8	0.128	9.4	0.107	-8.5	0.107	-8.5	0.162	38.5
	10	10	0.167	0.143	-14.4	0.180	7.8	0.159	-4.8	0.158	-5.4	0.192	15.0
		20	0.129	0.112	-13.2	0.138	7.0	0.123	-4.7	0.122	-5.4	0.160	24.0
		50	0.103	0.085	-17.5	0.108	4.9	0.094	-8.7	0.094	-8.7	0.135	31.1
	20	10	0.113	0.107	-5.3	0.119	5.3	0.113	0.0	0.112	-0.9	0.124	9.7
		20	0.091	0.082	-9.9	0.091	0.0	0.086	-5.5	0.086	-5.5	0.100	9.9
		50	0.072	0.065	-9.7	0.072	0.0	0.068	-5.6	0.068	-5.6	0.084	16.7
	30	10	0.094	0.087	-7.4	0.094	0.0	0.090	-4.3	0.090	-4.3	0.097	3.2
		20	0.073	0.068	-6.8	0.073	0.0	0.070	-4.1	0.070	-4.1	0.078	6.8
		50	0.055	0.053	-3.6	0.057	3.6	0.055	0.0	0.055	0.0	0.065	18.2

**WEB TABLE 4** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with CV = 0, when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.5$ . Number of cluster  $n \in \{8, 10, 20, 30\}$  and cluster size  $\bar{m} \in \{10, 20, 50\}$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

Parameter	$n$	$\bar{m}$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	8	10	0.809	0.876	0.899	<b>0.948</b>	0.852	0.919	0.845	0.916	0.865	0.928
		20	0.780	0.848	0.874	0.921	0.827	0.892	0.824	0.888	0.836	0.893
		50	0.769	0.844	0.869	0.923	0.819	0.886	0.817	0.884	0.820	0.892
	10	10	0.831	0.875	0.898	0.932	0.864	0.898	0.863	0.900	0.871	0.908
		20	0.824	0.872	0.895	0.926	0.852	0.899	0.855	0.899	0.869	0.908
		50	0.825	0.865	0.898	0.931	0.855	0.896	0.855	0.896	0.856	0.895
	20	10	0.903	0.917	<b>0.938</b>	<b>0.949</b>	0.916	0.931	0.917	0.931	0.916	0.929
		20	0.890	0.904	0.917	0.934	0.900	0.919	0.900	0.919	0.899	0.917
		50	0.901	0.918	0.930	<b>0.943</b>	0.917	0.932	0.917	0.932	0.915	0.929
	30	10	0.919	0.928	<b>0.943</b>	<b>0.949</b>	0.930	<b>0.943</b>	0.929	<b>0.943</b>	0.928	<b>0.937</b>
		20	0.890	0.906	0.920	0.932	0.908	0.919	0.908	0.918	0.905	0.917
		50	0.898	0.911	0.922	<b>0.936</b>	0.912	0.924	0.913	0.924	0.910	0.924
$\beta_{12}$	8	10	0.859	0.921	0.918	<b>0.961</b>	0.894	<b>0.939</b>	0.891	<b>0.941</b>	<b>0.963</b>	0.990
		20	0.844	0.910	0.909	<b>0.951</b>	0.878	0.926	0.878	0.924	0.965	0.987
		50	0.845	0.895	0.893	0.935	0.860	0.916	0.858	0.915	0.965	0.987
	10	10	0.875	0.915	0.919	<b>0.953</b>	0.897	<b>0.938</b>	0.895	<b>0.936</b>	<b>0.961</b>	0.982
		20	0.891	0.929	0.929	<b>0.955</b>	0.916	<b>0.945</b>	0.915	<b>0.943</b>	0.976	0.990
		50	0.876	0.917	0.916	<b>0.948</b>	0.892	0.928	0.891	0.927	0.984	0.993
	20	10	0.917	0.935	<b>0.937</b>	<b>0.950</b>	0.931	<b>0.943</b>	0.927	<b>0.939</b>	<b>0.958</b>	0.967
		20	0.906	0.926	0.927	<b>0.944</b>	0.918	<b>0.937</b>	0.917	0.933	<b>0.960</b>	0.974
		50	0.929	<b>0.951</b>	<b>0.953</b>	<b>0.962</b>	<b>0.942</b>	<b>0.957</b>	<b>0.939</b>	<b>0.956</b>	0.983	0.983
	30	10	<b>0.943</b>	<b>0.951</b>	<b>0.955</b>	<b>0.961</b>	<b>0.950</b>	<b>0.957</b>	<b>0.949</b>	<b>0.956</b>	<b>0.963</b>	0.967
		20	0.927	<b>0.942</b>	<b>0.944</b>	<b>0.954</b>	<b>0.937</b>	<b>0.949</b>	<b>0.936</b>	<b>0.947</b>	<b>0.963</b>	0.970
		50	0.930	<b>0.938</b>	<b>0.937</b>	<b>0.947</b>	0.935	<b>0.944</b>	<b>0.936</b>	<b>0.942</b>	0.973	0.983

**WEB TABLE 5** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under an unbalanced design with  $CV = 0.5$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.9$ . Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} \in \{10, 20, 50\}$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$n$	$\bar{m}$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	8	10	0.801	0.874	0.905	<b>0.946</b>	0.850	0.907	0.848	0.909	0.901	<b>0.962</b>
		20	0.795	0.871	0.906	<b>0.949</b>	0.853	0.911	0.847	0.907	0.884	0.935
		50	0.804	0.875	0.905	<b>0.941</b>	0.861	0.914	0.864	0.910	0.876	0.923
	10	10	0.806	0.878	0.912	<b>0.943</b>	0.865	0.917	0.862	0.919	0.894	<b>0.943</b>
		20	0.830	0.873	0.903	<b>0.944</b>	0.867	0.904	0.869	0.904	0.878	0.928
		50	0.815	0.882	0.916	<b>0.943</b>	0.866	0.914	0.866	0.914	0.880	0.919
	20	10	0.893	0.916	<b>0.940</b>	<b>0.951</b>	0.919	<b>0.938</b>	0.920	<b>0.937</b>	0.921	<b>0.937</b>
		20	0.894	0.907	0.934	<b>0.947</b>	0.906	0.930	0.905	0.931	0.908	0.930
		50	0.876	0.894	0.932	<b>0.951</b>	0.898	0.927	0.897	0.929	0.895	<b>0.956</b>
	30	10	0.915	0.928	<b>0.940</b>	<b>0.952</b>	0.931	<b>0.941</b>	0.930	<b>0.940</b>	0.929	<b>0.941</b>
		20	0.900	0.911	0.927	<b>0.944</b>	0.914	0.926	0.914	0.925	0.912	0.922
		50	0.912	0.927	<b>0.940</b>	<b>0.950</b>	0.929	<b>0.939</b>	0.929	<b>0.939</b>	0.927	0.933
$\beta_{12}$	8	10	0.863	0.917	0.917	<b>0.962</b>	0.890	<b>0.936</b>	0.890	0.934	<b>0.947</b>	0.984
		20	0.878	<b>0.940</b>	0.931	0.965	0.904	<b>0.951</b>	0.905	<b>0.951</b>	<b>0.956</b>	0.985
		50	0.863	0.928	0.918	<b>0.950</b>	0.899	<b>0.942</b>	0.898	<b>0.941</b>	<b>0.963</b>	0.988
	10	10	0.872	0.921	0.923	<b>0.957</b>	0.899	<b>0.941</b>	0.896	<b>0.938</b>	<b>0.948</b>	0.968
		20	0.860	0.920	0.921	<b>0.957</b>	0.890	<b>0.936</b>	0.892	<b>0.936</b>	<b>0.952</b>	0.982
		50	0.871	0.913	0.908	<b>0.941</b>	0.895	0.926	0.891	0.927	<b>0.949</b>	0.974
	20	10	0.916	0.932	<b>0.939</b>	<b>0.949</b>	0.927	<b>0.941</b>	0.925	<b>0.940</b>	<b>0.940</b>	<b>0.954</b>
		20	0.914	0.930	0.934	<b>0.942</b>	0.923	<b>0.938</b>	0.924	0.935	<b>0.936</b>	<b>0.956</b>
		50	0.898	0.925	0.932	<b>0.951</b>	0.913	<b>0.943</b>	0.915	<b>0.943</b>	<b>0.948</b>	<b>0.957</b>
	30	10	0.928	0.935	<b>0.939</b>	<b>0.947</b>	0.933	<b>0.940</b>	0.933	<b>0.940</b>	<b>0.938</b>	<b>0.948</b>
		20	0.935	<b>0.947</b>	<b>0.949</b>	<b>0.959</b>	<b>0.943</b>	<b>0.954</b>	<b>0.941</b>	<b>0.950</b>	<b>0.957</b>	<b>0.964</b>
		50	0.921	0.926	0.934	<b>0.944</b>	0.925	0.935	0.925	0.935	<b>0.940</b>	<b>0.955</b>

**WEB TABLE 6** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under an unbalanced design with  $CV = 0.5$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.5$ . Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} \in \{10, 20, 50\}$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$\bar{m}$	$n$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	8	10	0.776	0.849	0.892	0.934	0.832	0.892	0.832	0.889	0.842	0.908
		20	0.756	0.833	0.873	0.919	0.811	0.880	0.812	0.879	0.821	0.882
		50	0.772	0.854	0.879	0.923	0.825	0.892	0.822	0.889	0.831	0.887
	10	10	0.836	0.882	0.910	<b>0.936</b>	0.871	0.908	0.869	0.907	0.876	0.913
		20	0.800	0.862	0.892	0.927	0.842	0.900	0.839	0.896	0.844	0.903
		50	0.789	0.852	0.887	0.923	0.841	0.892	0.843	0.889	0.837	0.895
	20	10	0.857	0.883	0.908	0.924	0.885	0.904	0.886	0.905	0.883	0.903
		20	0.883	0.911	0.932	<b>0.941</b>	0.907	0.926	0.909	0.926	0.908	0.924
		50	0.870	0.892	0.913	0.930	0.897	0.914	0.897	0.914	0.889	0.910
	30	10	0.895	0.902	0.926	0.931	0.909	0.922	0.909	0.922	0.902	0.918
		20	0.902	0.911	0.930	<b>0.937</b>	0.913	0.926	0.913	0.926	0.911	0.921
		50	0.899	0.906	0.925	0.935	0.909	0.923	0.909	0.923	0.906	0.919
$\beta_{12}$	8	10	0.850	0.903	0.915	<b>0.951</b>	0.881	0.925	0.880	0.924	<b>0.949</b>	0.980
		20	0.816	0.889	0.897	<b>0.945</b>	0.854	0.920	0.855	0.919	<b>0.953</b>	0.987
		50	0.826	0.898	0.895	<b>0.941</b>	0.864	0.920	0.867	0.921	0.968	0.985
	10	10	0.883	0.926	0.933	0.969	0.903	<b>0.946</b>	0.899	<b>0.946</b>	0.964	0.986
		20	0.867	0.920	0.930	<b>0.957</b>	0.894	<b>0.942</b>	0.892	<b>0.938</b>	0.972	0.989
		50	0.855	0.903	0.916	<b>0.949</b>	0.885	<b>0.936</b>	0.885	0.931	0.964	0.978
	20	10	0.927	<b>0.948</b>	<b>0.958</b>	0.971	<b>0.941</b>	<b>0.959</b>	<b>0.939</b>	<b>0.957</b>	0.972	0.981
		20	0.901	0.919	0.923	0.935	0.911	0.930	0.912	0.927	<b>0.947</b>	<b>0.962</b>
		50	0.904	0.923	0.926	<b>0.942</b>	0.916	0.930	0.915	0.930	<b>0.964</b>	0.975
	30	10	0.919	0.932	<b>0.939</b>	<b>0.951</b>	0.931	<b>0.942</b>	0.925	<b>0.941</b>	0.915	0.927
		20	0.915	0.927	<b>0.936</b>	<b>0.945</b>	0.924	<b>0.938</b>	0.925	<b>0.936</b>	<b>0.957</b>	0.968
		50	0.924	<b>0.937</b>	<b>0.941</b>	<b>0.950</b>	<b>0.937</b>	<b>0.942</b>	0.935	<b>0.940</b>	0.965	0.969

**WEB TABLE 7** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.9$ . Number of cluster  $n \in \{40, 60, 80, 100, 120\}$  and cluster size  $\bar{m} \in \{10, 20\}$ .

$\beta$	$\bar{m}$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
				ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	10	40	0.108	0.097	-10.2	0.106	-1.9	0.101	-6.5	0.101	-6.5	0.101	-6.5
		60	0.091	0.085	-6.6	0.092	1.1	0.088	-3.3	0.088	-3.3	0.088	-3.3
		80	0.083	0.081	-2.4	0.086	3.6	0.083	0.0	0.083	0.0	0.083	0.0
		100	0.073	0.069	-5.5	0.073	0.0	0.071	-2.7	0.071	-2.7	0.071	-2.7
		120	0.074	0.069	-6.8	0.072	-2.7	0.071	-4.1	0.071	-4.1	0.071	-4.1
	20	40	0.063	0.059	-6.3	0.062	-1.6	0.060	-4.8	0.060	-4.8	0.060	-4.8
		60	0.063	0.062	-1.6	0.064	1.6	0.063	0.0	0.063	0.0	0.063	0.0
		80	0.054	0.054	0.0	0.055	1.9	0.054	0.0	0.054	0.0	0.054	0.0
		100	0.061	0.060	-1.6	0.061	0.0	0.060	-1.6	0.060	-1.6	0.060	-1.6
		120	0.048	0.050	4.2	0.051	6.2	0.050	4.2	0.050	4.2	0.050	4.2
$\beta_{12}$	10	40	0.078	0.074	-5.1	0.077	-1.3	0.076	-2.6	0.075	-3.8	0.078	0.0
		60	0.055	0.054	-1.8	0.056	1.8	0.055	0.0	0.055	0.0	0.057	3.6
		80	0.064	0.060	-6.3	0.062	-3.1	0.061	-4.7	0.061	-4.7	0.062	-3.1
		100	0.047	0.044	-6.4	0.046	-2.1	0.045	-4.3	0.045	-4.3	0.046	-2.1
		120	0.054	0.052	-3.7	0.054	0.0	0.053	-1.9	0.053	-1.9	0.054	0.0
	20	40	0.038	0.038	0.0	0.039	2.6	0.039	2.6	0.039	2.6	0.039	2.6
		60	0.046	0.047	2.2	0.048	4.3	0.047	2.2	0.047	2.2	0.048	4.3
		80	0.036	0.034	-5.6	0.035	-2.8	0.035	-2.8	0.035	-2.8	0.035	-2.8
		100	0.043	0.043	0.0	0.044	2.3	0.043	0.0	0.043	0.0	0.044	2.3
		120	0.033	0.032	-3.0	0.032	-3.0	0.032	-3.0	0.032	-3.0	0.032	-3.0

**WEB TABLE 8** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under an unbalanced design with  $CV = 0.5$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.5$ . Number of cluster  $n \in \{40, 60, 80, 100, 120\}$  and average cluster size  $\bar{m} \in \{10, 20\}$ .

$\beta$	$\bar{m}$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
				ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	10	40	0.182	0.165	-9.3	0.182	0.0	0.173	-4.9	0.173	-4.9	0.170	-6.6
		60	0.174	0.159	-8.6	0.175	0.6	0.167	-4.0	0.167	-4.0	0.164	-5.7
		80	0.147	0.135	-8.2	0.145	-1.4	0.140	-4.8	0.140	-4.8	0.138	-6.1
		100	0.146	0.133	-8.9	0.142	-2.7	0.137	-6.2	0.137	-6.2	0.135	-7.5
		120	0.122	0.118	-3.3	0.124	1.6	0.121	-0.8	0.121	-0.8	0.119	-2.5
	20	40	0.122	0.116	-4.9	0.122	0.0	0.119	-2.5	0.119	-2.5	0.118	-3.3
		60	0.110	0.106	-3.6	0.110	0.0	0.108	-1.8	0.108	-1.8	0.107	-2.7
		80	0.109	0.103	-5.5	0.108	-0.9	0.105	-3.7	0.105	-3.7	0.104	-4.6
		100	0.100	0.096	-4.0	0.100	0.0	0.098	-2.0	0.098	-2.0	0.097	-3.0
		120	0.097	0.095	-2.1	0.098	1.0	0.096	-1.0	0.096	-1.0	0.096	-1.0
$\beta_{12}$	10	40	0.077	0.075	-2.6	0.079	2.6	0.077	0.0	0.076	-1.3	0.081	5.2
		60	0.063	0.059	-6.3	0.062	-1.6	0.061	-3.2	0.061	-3.2	0.066	4.8
		80	0.065	0.061	-6.2	0.064	-1.5	0.062	-4.6	0.062	-4.6	0.065	0.0
		100	0.049	0.048	-2.0	0.050	2.0	0.049	0.0	0.049	0.0	0.052	6.1
		120	0.055	0.054	-1.8	0.055	0.0	0.055	0.0	0.054	-1.8	0.056	1.8
	20	40	0.041	0.042	2.4	0.043	4.9	0.043	4.9	0.043	4.9	0.045	9.8
		60	0.049	0.048	-2.0	0.049	0.0	0.049	0.0	0.049	0.0	0.050	2.0
		80	0.038	0.038	0.0	0.039	2.6	0.038	0.0	0.038	0.0	0.040	5.3
		100	0.043	0.044	2.3	0.045	4.7	0.044	2.3	0.044	2.3	0.045	4.7
		120	0.034	0.034	0.0	0.035	2.9	0.035	2.9	0.035	2.9	0.036	5.9



**WEB TABLE 9** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.9$ . Number of cluster  $n \in \{40, 60, 80, 100, 120\}$  and cluster size  $\bar{m} \in \{10, 20\}$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$\bar{m}$	$n$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	10	40	0.930	<b>0.940</b>	<b>0.947</b>	<b>0.955</b>	<b>0.941</b>	<b>0.945</b>	<b>0.941</b>	<b>0.944</b>	<b>0.940</b>	<b>0.944</b>
		60	<b>0.937</b>	<b>0.941</b>	<b>0.947</b>	<b>0.951</b>	<b>0.939</b>	<b>0.946</b>	<b>0.939</b>	<b>0.944</b>	<b>0.941</b>	<b>0.946</b>
		80	0.933	<b>0.937</b>	<b>0.943</b>	<b>0.946</b>	<b>0.939</b>	<b>0.941</b>	<b>0.939</b>	<b>0.941</b>	<b>0.937</b>	<b>0.949</b>
		100	<b>0.948</b>	<b>0.950</b>	<b>0.953</b>	<b>0.957</b>	<b>0.951</b>	<b>0.953</b>	<b>0.950</b>	<b>0.952</b>	<b>0.951</b>	<b>0.953</b>
		120	<b>0.947</b>	<b>0.949</b>	<b>0.955</b>	<b>0.956</b>	<b>0.948</b>	<b>0.949</b>	<b>0.948</b>	<b>0.950</b>	<b>0.950</b>	<b>0.951</b>
	20	40	0.914	0.926	<b>0.944</b>	<b>0.946</b>	0.927	<b>0.941</b>	0.927	<b>0.941</b>	0.927	<b>0.940</b>
		60	0.928	<b>0.936</b>	<b>0.944</b>	<b>0.954</b>	<b>0.938</b>	<b>0.944</b>	<b>0.938</b>	<b>0.944</b>	<b>0.936</b>	<b>0.942</b>
		80	0.932	<b>0.938</b>	<b>0.940</b>	<b>0.946</b>	<b>0.936</b>	<b>0.942</b>	<b>0.938</b>	<b>0.942</b>	<b>0.938</b>	<b>0.943</b>
		100	<b>0.938</b>	<b>0.942</b>	<b>0.946</b>	<b>0.950</b>	<b>0.944</b>	<b>0.946</b>	<b>0.942</b>	<b>0.946</b>	<b>0.942</b>	<b>0.946</b>
		120	<b>0.950</b>	<b>0.954</b>	<b>0.952</b>	<b>0.960</b>	<b>0.951</b>	<b>0.955</b>	<b>0.951</b>	<b>0.955</b>	<b>0.951</b>	<b>0.953</b>
$\beta_{12}$	10	40	0.928	<b>0.940</b>	<b>0.942</b>	<b>0.948</b>	<b>0.937</b>	<b>0.943</b>	<b>0.937</b>	<b>0.943</b>	<b>0.944</b>	<b>0.947</b>
		60	0.930	<b>0.937</b>	<b>0.943</b>	<b>0.950</b>	<b>0.936</b>	<b>0.944</b>	0.935	<b>0.943</b>	<b>0.941</b>	<b>0.952</b>
		80	<b>0.936</b>	<b>0.939</b>	<b>0.940</b>	<b>0.942</b>	<b>0.938</b>	<b>0.942</b>	<b>0.937</b>	<b>0.940</b>	<b>0.941</b>	<b>0.944</b>
		100	<b>0.951</b>	<b>0.956</b>	<b>0.956</b>	<b>0.959</b>	<b>0.955</b>	<b>0.958</b>	<b>0.955</b>	<b>0.957</b>	<b>0.958</b>	<b>0.959</b>
		120	<b>0.950</b>	<b>0.953</b>	<b>0.953</b>	<b>0.954</b>	<b>0.952</b>	<b>0.953</b>	<b>0.952</b>	<b>0.953</b>	<b>0.953</b>	<b>0.955</b>
	20	40	<b>0.941</b>	<b>0.945</b>	<b>0.947</b>	<b>0.954</b>	<b>0.944</b>	<b>0.947</b>	<b>0.943</b>	<b>0.947</b>	<b>0.950</b>	<b>0.959</b>
		60	0.934	<b>0.936</b>	<b>0.939</b>	<b>0.948</b>	<b>0.936</b>	<b>0.939</b>	<b>0.936</b>	<b>0.939</b>	<b>0.938</b>	<b>0.950</b>
		80	<b>0.946</b>	<b>0.946</b>	<b>0.948</b>	<b>0.952</b>	<b>0.946</b>	<b>0.950</b>	<b>0.946</b>	<b>0.950</b>	<b>0.952</b>	<b>0.954</b>
		100	<b>0.936</b>	<b>0.944</b>	<b>0.946</b>	<b>0.952</b>	<b>0.946</b>	<b>0.946</b>	<b>0.946</b>	<b>0.946</b>	<b>0.946</b>	<b>0.950</b>
		120	<b>0.940</b>	<b>0.944</b>	<b>0.946</b>	<b>0.946</b>	<b>0.942</b>	<b>0.946</b>	<b>0.942</b>	<b>0.946</b>	<b>0.946</b>	<b>0.946</b>

**WEB TABLE 10** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under an unbalanced design with  $CV = 0.5$ , when the cluster-level frailty is generated from the standard positive stable distribution with  $\alpha = 0.5$ . Number of cluster  $n \in \{40, 60, 80, 100, 120\}$  and cluster size  $\bar{m} \in \{10, 20\}$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$\bar{m}$	$n$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	10	40	0.914	0.921	<b>0.938</b>	<b>0.945</b>	0.923	<b>0.939</b>	0.922	<b>0.938</b>	0.920	0.931
		60	0.930	0.932	<b>0.949</b>	<b>0.951</b>	<b>0.937</b>	<b>0.945</b>	<b>0.937</b>	<b>0.944</b>	0.932	<b>0.941</b>
		80	0.929	0.931	<b>0.938</b>	<b>0.941</b>	0.934	<b>0.938</b>	0.934	<b>0.937</b>	0.931	<b>0.936</b>
		100	0.931	0.934	<b>0.938</b>	<b>0.944</b>	0.934	0.935	0.934	0.935	0.934	0.935
		120	0.933	0.935	<b>0.937</b>	<b>0.941</b>	<b>0.936</b>	<b>0.937</b>	<b>0.936</b>	<b>0.937</b>	0.935	<b>0.937</b>
	20	40	0.911	0.922	0.935	<b>0.942</b>	0.924	0.933	0.924	0.933	0.921	0.932
		60	0.914	0.920	0.928	0.932	0.918	0.926	0.918	0.926	0.920	0.924
		80	0.929	0.931	<b>0.938</b>	<b>0.941</b>	0.934	<b>0.938</b>	0.934	<b>0.937</b>	0.931	<b>0.936</b>
		100	<b>0.938</b>	<b>0.938</b>	<b>0.944</b>	<b>0.946</b>	<b>0.940</b>	<b>0.944</b>	<b>0.940</b>	<b>0.944</b>	<b>0.938</b>	<b>0.942</b>
		120	0.932	0.934	<b>0.940</b>	<b>0.946</b>	<b>0.936</b>	<b>0.940</b>	<b>0.936</b>	<b>0.940</b>	0.934	<b>0.936</b>
$\beta_{12}$	10	40	0.934	<b>0.944</b>	<b>0.949</b>	<b>0.951</b>	<b>0.943</b>	<b>0.950</b>	<b>0.940</b>	<b>0.950</b>	<b>0.958</b>	<b>0.964</b>
		60	0.932	<b>0.936</b>	<b>0.942</b>	<b>0.948</b>	<b>0.937</b>	<b>0.942</b>	<b>0.936</b>	<b>0.941</b>	<b>0.950</b>	<b>0.956</b>
		80	<b>0.938</b>	<b>0.940</b>	<b>0.945</b>	<b>0.950</b>	<b>0.939</b>	<b>0.943</b>	<b>0.939</b>	<b>0.942</b>	<b>0.950</b>	<b>0.955</b>
		100	<b>0.944</b>	<b>0.946</b>	<b>0.949</b>	<b>0.949</b>	<b>0.946</b>	<b>0.949</b>	<b>0.945</b>	<b>0.949</b>	<b>0.950</b>	<b>0.954</b>
		120	<b>0.943</b>	<b>0.944</b>	<b>0.948</b>	<b>0.949</b>	<b>0.944</b>	<b>0.948</b>	<b>0.944</b>	<b>0.948</b>	<b>0.948</b>	<b>0.949</b>
	20	40	0.922	0.932	<b>0.936</b>	<b>0.947</b>	0.931	<b>0.938</b>	0.929	<b>0.937</b>	<b>0.951</b>	<b>0.957</b>
		60	<b>0.936</b>	<b>0.944</b>	<b>0.944</b>	<b>0.952</b>	<b>0.944</b>	<b>0.944</b>	<b>0.942</b>	<b>0.944</b>	<b>0.956</b>	<b>0.956</b>
		80	0.934	<b>0.944</b>	<b>0.950</b>	<b>0.950</b>	<b>0.946</b>	<b>0.950</b>	<b>0.942</b>	<b>0.946</b>	<b>0.950</b>	<b>0.958</b>
		100	<b>0.942</b>	<b>0.944</b>	<b>0.946</b>	<b>0.952</b>	<b>0.944</b>	<b>0.946</b>	<b>0.944</b>	<b>0.946</b>	<b>0.954</b>	<b>0.958</b>
		120	<b>0.942</b>	<b>0.952</b>	<b>0.954</b>	<b>0.954</b>	<b>0.952</b>	<b>0.952</b>	<b>0.950</b>	<b>0.954</b>	<b>0.954</b>	<b>0.956</b>

**WEB TABLE 11** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the log-normal distribution with zero mean parameter and standard deviation parameter 0.5 (both on log scale). Number of cluster  $n \in \{8, 10, 20, 30\}$  and cluster size  $\bar{m} = 10$ .

Parameter	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	8	0.318	0.211	-33.6	0.328	3.1	0.252	-20.8	0.249	-21.7	0.265	-16.7
	10	0.253	0.188	-25.7	0.259	2.4	0.216	-14.6	0.215	-15.0	0.223	-11.9
	20	0.166	0.137	-17.5	0.161	-3.0	0.148	-10.8	0.148	-10.8	0.148	-10.8
	30	0.125	0.112	-10.4	0.125	0.0	0.118	-5.6	0.118	-5.6	0.117	-6.4
$\beta_{12}$	8	0.174	0.153	-12.1	0.189	8.6	0.169	-2.9	0.168	-3.4	0.201	15.5
	10	0.154	0.138	-10.4	0.163	5.8	0.149	-3.2	0.148	-3.9	0.170	10.4
	20	0.112	0.099	-11.6	0.108	-3.6	0.104	-7.1	0.103	-8.0	0.110	-1.8
	30	0.084	0.083	-1.2	0.089	6.0	0.086	2.4	0.085	-5.6	0.089	6.0



**WEB TABLE 12** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with CV = 0, when the cluster-level frailty is generated from the log-normal distribution with zero mean parameter and standard deviation parameter 0.9 (both on log scale). Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} = 10$ .

$\beta$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	8	0.429	0.270	-37.1	0.430	0.2	0.323	-24.7	0.320	-25.4	0.329	-23.3
	10	0.349	0.256	-26.6	0.369	5.7	0.297	-14.9	0.295	-15.5	0.296	-15.2
	20	0.205	0.178	-13.2	0.210	2.4	0.193	-5.9	0.192	-6.3	0.191	-6.8
	30	0.163	0.147	-9.8	0.166	1.8	0.156	-4.3	0.155	-4.9	0.154	-5.5
$\beta_{12}$	8	0.193	0.151	-21.8	0.188	-2.6	0.167	-13.5	0.166	-14.0	0.206	6.7
	10	0.164	0.137	-16.5	0.164	0.0	0.149	-9.1	0.147	-10.4	0.177	7.9
	20	0.110	0.101	-8.2	0.111	0.9	0.106	-3.6	0.105	-4.5	0.115	4.5
	30	0.089	0.084	-5.6	0.090	1.1	0.087	-2.2	0.086	-3.4	0.091	2.2

**WEB TABLE 13** Empirical standard errors (ESE) and average estimated standard errors (ASE) for the regression coefficients, and percent of relative biases (BIAS) using the uncorrected sandwich ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with CV = 0, when the cluster-level frailty is generated from the log-normal distribution with zero mean parameter and standard deviation parameter 1.5 (both on log scale). Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} = 10$ .

$\beta$	$n$	ESE	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
			ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS	ASE	BIAS
$\beta_{11}$	8	0.541	0.343	-36.6	0.617	14.0	0.418	-22.7	0.407	-24.8	0.411	-24.0
	10	0.425	0.308	-27.5	0.429	0.9	0.356	-16.2	0.353	-16.9	0.353	-16.9
	20	0.258	0.220	-14.7	0.258	0.0	0.237	-8.1	0.237	-8.1	0.234	-9.3
	30	0.209	0.180	-13.9	0.202	-3.3	0.190	-9.1	0.190	-9.1	0.188	-10.0
$\beta_{12}$	8	0.187	0.157	-16.0	0.212	13.4	0.178	-4.8	0.175	-6.4	0.225	20.3
	10	0.168	0.141	-16.1	0.171	1.8	0.154	-8.3	0.152	-9.5	0.189	12.5
	20	0.113	0.104	-8.0	0.114	0.9	0.109	-3.5	0.108	-4.4	0.121	7.1
	30	0.091	0.086	-5.5	0.092	1.1	0.089	-2.2	0.088	-3.3	0.096	5.5

**WEB TABLE 14** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the log-normal distribution with zero mean parameter and standard deviation parameter 0.5 (both on log scale). Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} = 10$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$n$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
		$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	8	0.780	0.869	0.884	<b>0.939</b>	0.833	0.903	0.834	0.903	0.888	<b>0.948</b>
	10	0.823	0.872	0.894	<b>0.937</b>	0.858	0.906	0.858	0.908	0.887	0.932
	20	0.875	0.896	0.915	<b>0.938</b>	0.891	0.913	0.891	0.912	0.902	0.920
	30	0.902	0.919	0.935	<b>0.943</b>	0.923	0.934	0.923	0.934	0.921	0.931
$\beta_{12}$	8	0.894	<b>0.946</b>	<b>0.942</b>	0.970	0.920	<b>0.962</b>	0.919	<b>0.957</b>	0.966	0.987
	10	0.908	<b>0.938</b>	<b>0.937</b>	<b>0.956</b>	0.923	<b>0.945</b>	0.923	<b>0.943</b>	<b>0.955</b>	0.971
	20	0.898	0.922	0.926	<b>0.943</b>	0.912	0.933	0.909	0.927	0.933	<b>0.952</b>
	30	<b>0.944</b>	<b>0.957</b>	<b>0.960</b>	0.965	<b>0.952</b>	<b>0.963</b>	<b>0.950</b>	<b>0.963</b>	<b>0.963</b>	0.967

**WEB TABLE 15** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the log-normal distribution with zero mean parameter and standard deviation parameter 0.9 (both on log scale). Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} = 10$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$\bar{m}$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
		$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	8	0.770	0.858	0.881	<b>0.936</b>	0.822	0.899	0.825	0.900	0.863	0.923
	10	0.835	0.890	0.920	<b>0.949</b>	0.881	0.922	0.879	0.922	0.891	0.933
	20	0.874	0.902	0.917	<b>0.939</b>	0.902	0.915	0.898	0.915	0.901	0.914
	30	0.916	0.921	0.934	<b>0.943</b>	0.926	0.935	0.926	0.933	0.921	0.930
$\beta_{12}$	8	0.859	0.921	0.915	<b>0.957</b>	0.886	<b>0.938</b>	0.884	0.935	<b>0.958</b>	0.982
	10	0.876	0.924	0.925	<b>0.955</b>	0.901	<b>0.943</b>	0.898	<b>0.943</b>	<b>0.954</b>	0.981
	20	0.920	<b>0.943</b>	<b>0.946</b>	<b>0.961</b>	0.932	<b>0.950</b>	0.927	<b>0.948</b>	<b>0.961</b>	0.973
	30	0.935	<b>0.942</b>	<b>0.943</b>	<b>0.948</b>	<b>0.940</b>	<b>0.945</b>	<b>0.940</b>	<b>0.945</b>	<b>0.947</b>	<b>0.956</b>

**WEB TABLE 16** Empirical coverage percentage of nominal 95% confidence intervals based upon standard normal distribution ( $z$ ) and  $t$ -distribution quantiles ( $t$ ) based on the uncorrected sandwich estimator ( $\hat{V}_{UC}$ ), and four bias-corrected variance estimators ( $\hat{V}_{MD}$ ,  $\hat{V}_{KC}$ ,  $\hat{V}_{FG}$ ,  $\hat{V}_{MBN}$ ) under a balanced design with  $CV = 0$ , when the cluster-level frailty is generated from the log-normal distribution with zero mean parameter and standard deviation parameter 1.5 (both on log scale). Number of cluster  $n \in \{8, 10, 20, 30\}$  and average cluster size  $\bar{m} = 10$ . Empirical coverage percentages between 0.936 and 0.964 are considered close to nominal and highlighted in bold font.

$\beta$	$n$	$\hat{V}_{UC}$		$\hat{V}_{MD}$		$\hat{V}_{KC}$		$\hat{V}_{FG}$		$\hat{V}_{MBN}$	
		$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$	$z$	$t$
$\beta_{11}$	8	0.776	0.844	0.872	<b>0.936</b>	0.831	0.893	0.822	0.892	0.840	0.916
	10	0.829	0.888	0.905	<b>0.939</b>	0.871	0.913	0.870	0.912	0.882	0.926
	20	0.887	0.905	0.918	<b>0.940</b>	0.902	0.921	0.902	0.917	0.903	0.924
	30	0.900	0.917	0.931	<b>0.938</b>	0.916	0.931	0.914	0.931	0.917	0.927
$\beta_{12}$	8	0.871	0.920	0.924	<b>0.962</b>	0.898	<b>0.945</b>	0.896	<b>0.941</b>	<b>0.964</b>	0.988
	10	0.871	0.928	0.932	<b>0.961</b>	0.908	<b>0.942</b>	0.900	<b>0.940</b>	0.966	0.980
	20	0.925	<b>0.937</b>	<b>0.941</b>	<b>0.957</b>	0.931	<b>0.944</b>	0.931	<b>0.941</b>	<b>0.960</b>	0.969
	30	<b>0.936</b>	<b>0.945</b>	<b>0.955</b>	<b>0.960</b>	<b>0.942</b>	<b>0.954</b>	<b>0.940</b>	<b>0.951</b>	<b>0.959</b>	0.966